Class Notes on Housing Market Analysis

PPA 786
Urban Policy
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Introduction

Analysis of urban problems and urban policy requires an understanding of urban housing markets. In fact, because urban problems virtually all have a spatial dimension and because housing markets determine the distribution of different types of people across urban space, housing market analysis forms a foundation for thinking about most urban policies. The key concepts and analytical tools can be found in many urban economics textbooks (such as Arthur O'Sullivan, *Urban Economics*, 5th Edition, McGraw-Hill, 2003), but the presentation in these books is more technical than necessary and does not draw out several features of urban housing markets that are crucial for this course. These notes are designed to provide basic concepts and simple analytical tools that make it possible to understand urban housing markets and to bring the spatial dimension into an evaluation of any urban policy.

These notes are organized in three parts. The first part presents basic concepts of land and housing markets and develops the principal analytical tool for housing market analysis, which is called a bid function. The second part develops the concept of household sorting, which is the market process that allocates different types of households to different locations in an urban area. This concept has many important applications in urban policy; for example, it helps to explain concentrated poverty. The second part also introduces the concept of neighborhood amenities, which are crucial both for household sorting and for neighborhood change. The third part of the notes shows how bid functions and household sorting concepts can be used to help understand both neighborhood change and long-run equilibrium in an urban area.

Part I: Land Concepts, Housing Concepts, and Household Bid Functions

Any discussion of housing must begin with land, which is a key input into housing and in fact is the input that gives housing a spatial dimension. Moreover, land concepts provide a simple entry into housing market concepts. This part of the notes presents land concepts, then turns to housing concepts, and finally to bid functions.

The Land Market

The first concepts are land rent and land value, which are defined as follows:

Land rent is the price for using one unit of land, say an acre, for one unit of time, say a year.

Land value is the price of buying one unit of land, again say an acre.

The first analytical question to ask about land is: How are these two concepts related? The answer to this question flows from another question: How much would an investor be willing to pay to purchase one acre of land? The answer is that she would be willing to pay the present value of the stream of rents (net of expenses) that the owner of the land would receive.[(1)] In other words, land value equals the discounted value of the stream of annual net land rents. This type of relationship holds for the price of any asset; that is, the price of any asset, including land, equals the present value of the stream of annual (or monthly or weekly or daily) benefits from owning it.

In symbols, let R_{Lt} stand for land rent in year t, V_L for land value, i for the discount rate. Then, assuming that payments are received at the end of each period, the relationship between V_L and R_L can be written as follows:

$$V_L = \frac{R_{L1}}{(1+i)} + \frac{R_{L2}}{(1+i)^2} + \frac{R_{L3}}{(1+i)^3} + \frac{R_{L4}}{(1+i)^4} + \ldots = \sum_{t=1}^{\infty} \frac{R_{Lt}}{(1+i)^t} \quad (1)$$

In the important special case that R_{Lt} is a constant over time, so that the time subscript can be dropped, this formula simplifies to:[(2)]

$$V_L = \frac{R_L}{i} \quad (2)$$

Remember that all discounting formulas must be consistent in the treatment of inflation. If R_L is constant in nominal terms (perhaps due to a long-term contract) then a nominal interest rate must be used for i. In the more likely case that R_L is constant in real terms, i must be a real interest rate; that is, it must equal an observed market interest rate minus the anticipated rate of inflation.

Because land is an input into the production of many things, including housing, a distinction often is made between two land value concepts:

Unimproved land value is the value of land with nothing on it, or the value of the land itself, excluding the value of any structures that may be on it.

Improved land value is the value of the land itself plus the value of the structure built on it.

Unimproved land value is a tricky concept both to think about and to measure. It is difficult to think about because the amount someone is willing to pay for land includes the net return (that is, the return after construction costs) from buildings that could be built on it. The relevant buildings here are the ones that yield the highest return on a particular piece of land. The point here is that land has no intrinsic value, it only has value as an input into the production of something.[(3)]

Improved land value is the same thing as **property value**. For example, the value of a single-family house, which is considered in detail later on, is an example of an improved land value.

One implication of these definitions is that the difference between the improved and unimproved value of a particular piece of land equals the cost of constructing the property that will yield the highest return. After all, it is the construction of this property that translates the hypothetical return in the unimproved land value concept into the actual return in the improved land value concept. In other words, the construction cost is what an investor saves if she buys land with improvements already on it.

In the first section of these notes, we will focus on unimproved land value. This value often is difficult to observe because most urban areas do not have a great deal of vacant land--let alone frequent sales of vacant land. In other words, most actual land sales involve land with improvements, so the unimproved land value must be inferred (by subtracting estimated construction costs or by some other method).

Land is an input that goes directly into producing housing, but it also appears in the production of almost everything else. Factories and stores and restaurants and theaters all must occupy land, so businesses must decide how much land to buy or rent, just as they must decide about capital and labor. Recall from basic microeconomics that the demand for an input is a **derived demand**, which equals the value of the input's marginal product multiplied by the price of the output being produced. In other words, the amount a firm is willing to pay for another unit of the input equals the money it will earn when that input is purchased, which equals the quantity of product that additional unit will produce multiplied by the revenue that quantity of product will generate in the marketplace. The horizontal sum of the demand curves of individual firms equals the market demand for the product.

The market supply of an input typically depends on the behavior of the owners of that input. In the case of land, however, the supply is approximately fixed. For most applications, therefore, the overall supply curve for land can be treated as vertical. In some cases, the possibility of creating usable land from landfill may move the supply curve somewhat away from vertical. Moreover, the supply of land for a particular type of activity is not vertical in general because land can be moved from one use to another.

In competitive markets (on which our attention will focus), all actors are price takers. Hence no individual can affect the market price of an input. The price of land is found at the intersection of the (derived) demand curve and the (vertical) supply curve, and each user of land adjusts the quantity of land she rents (or buys) so that the value of her marginal product equals the market land rent (or value).

Some people have argued that land is different than other inputs because its total supply is fixed. For example, two famous early economists, David Ricardo and Henry George, argued that land rents will absorb all the fruits of economic progress. Since land supply is fixed and all products must use land as an input, the argument goes, the owners of land will simply boost rents until they claim all the returns to productive activity in society.

This argument turns out to be incorrect. With competitive markets (and there is no reason to believe that the land market is not usually fairly competitive), all inputs, including land, are paid the value of their marginal product. Moreover, in the long run firms all produce at the minimum point on their average cost curve where product price equals average cost equals marginal cost. Hence average revenue (i.e. price) just equals average cost; that is, payments to inputs just exhaust total revenue. In this context, all inputs play a symmetrical role and there is no basis for singling out land, or any other input, as different from the others.

Henry George took this argument a step further and said that land rents should be taxed away. After all, he said, land rents are not based on work effort or investment in capital or new technology. Moreover, since the supply of land is fixed, there will be no economic distortion from eliminating its return. Finally, land tends to be owned by rich people, who should, many people say, be taxed more heavily than others. In fact, George went so far as to propose switching to a single tax on land.

If you believe in progressive taxation, there is some truth to the view that a tax on land would be fairer than some other taxes. However, most experts prefer a progressive tax on income, which is a better measure of a person's ability to pay than is the value of the land they own.

Moreover, George's efficiency arguments contain a fatal flaw; they miss the fact that land rents are signals that determine what type of economic activity takes place at each location. We will explore these signals at length later in these notes. We will discover that different activities compete for access to each location and the highest bidder, who obviously wins the competition, is the one who gets the most benefit from locating there. If you tax away land rents, you eliminate these signals and make it impossible for land markets to allocate activities. Eliminating land rents therefore would cause major distortions in the land market--and in all markets that use land as inputs.

There remains a debate, however, about whether land should be taxed at a higher rate than improvements. A few places, including the city of Pittsburgh, levy higher property tax rates on land than on buildings. In other words, a Pittsburgh-type policy levies a high tax rate on the estimated unimproved land value and a low rate on its estimate of the difference between improved and unimproved land value. This difference, you will recall, equals construction costs, so this approach can be through of as a construction subsidy. A few academics argue that this subsidy encourages investment in plant and equipment and thereby encourages local economic development. The problem with this argument is that, even if one could estimate unimproved land value accurately, it is not clear that subsidizing construction has a greater impact on local economic development than subsidizing any number of other activities. So far there is no good evidence to resolve this debate one way or the other. Because the debate involves complex economic models, it will not be pursued further here. Suffice it to say that for the most part, land is singled out here and in other courses on urban policy or urban economics because it is linked to the spatial allocation of households and firms, not because it is conceptually different from other inputs.

These land concepts allow us to introduce the notion of a **bid function** in a very simple model without housing. A bid function for housing will be developed later in these notes. The simple model presented here is designed for pedagogical purposes and is not intended to be realistic.

Imagine an urban area with a central export point, such as an airport or a rail terminal. Firms in the area produce a single product and ship it to the export point, where it is shipped out for sale. The transportation system will be simplified by assuming that the product can be transported in a straight line from any factory to the export point at a cost of *s* dollars per unit per mile. These assumptions imply that all locations a given distance from the export node are equivalent because they each involve the same total transportation costs (and because nothing else about location matters in this simple model). The price of the product is P_Q. Thus if a factor is located u miles away from the export point, the net price of the product (that is, the price net of transportation costs) is $(P_Q - su)$.

Now suppose identical firms produce the product using land and other inputs and that the marginal product of land is a constant, a.[(4)] A marginal product equal to a implies that a one-unit increase in land results in an a-unit increase in the product. Equivalently, to obtain a one-unit increase in the product, the firm must purchase $1/a$ units of land. The value of the marginal product of land equals the marginal product multiplied by the net product price or $a(P_Q - su)$. Moreover, a competitive firm sets the value of the marginal product of each input equal to the price of the input. Hence the land component of this model can be summarized by the following equation:

$$a(P_Q - su) = R_L(u) \quad , \qquad (3)$$

where $R_L(u)$ is land rent at a location u miles from the export point. In this equation, a is set by technology, P_Q is set by competition in the national market, and s is set by the transportation system. Thus, at a given location, that is, at a given value of u, everything on the left side of this equation is known. In other words, this equation determines the pattern of land rents, $R_L(u)$.

Now comes the tricky part: interpreting $R_L(u)$. We have shown what land rents must be to ensure that the value of the marginal product of land equals the land rent. As it turns out, however, $R_L(u)$ has another, somewhat more abstract interpretation. In particular, it is the maximum amount a firm would be willing to pay for a unit of land at location u, or to use the standard term, it is the firm's bid function for land. This terminology reflects the fact that firms must bid against each other for land. In a competitive market, all firms must make zero economic profits. Thus firms will not locate far from the export point unless they are compensated for the resulting high cost of transporting their product. The bid function indicates how much the price of land must drop as one moves away from the export point in order to compensate firms for their higher transportation costs. If $R_L(u)$ did not take the form given by equation (3), some locations would be more profitable than others, firms would compete against each other for access to those locations, and land rents at those locations would rise. Equation (3) describes the only possible equilibrium land-rent function. In other words, equation (3) describes a set of land rents that makes (identical) firms indifferent to their location. Firms do not care whether they are close to or far from the export point because they are exactly compensated for high transportation costs in the form of low land rents.

The simple bid function described by equation (3) is illustrated in panel A of Figure 1. In this simple model, land rent is a linear function of distance from the export point. The term (aP_Q) is the intercept of the line and the term ($-sa$) is the slope. Thus land rent is a downward sloping linear function of u with a slope of ($-sa$). If a firm moves one mile farther from the city, its transportation costs per unit of output go up by \$$s$ dollars. The firm uses ($1/a$) units of land to produce one unit of its product. Thus this increase in transportation costs is exactly offset by the decline in land rent multiplied by the amount of land required per unit of product: $(-sa)(1/a) = -\$s$.

This bid function is linear because it does not allow for *substitution* between land and capital. In a more realistic model, firms would substitute away from land toward capital at locations where the price of land is high, namely locations close to the export point. Because firms consume less land near the export point, a greater drop in the price of land is needed there to compensate them for a given increase in transportation costs. Introducing substitution into the model therefore results in bid functions that are steeper close to an export point than they are far from the export point. A bid function of this type is illustrated in panel B of Figure 1.

The Housing Market

In the basic microeconomic model of household behavior, a household chooses, from among the combinations it can afford, the combination of commodities that maximizes its utility. Urban economics, on which these notes draw, recognizes that a household also chooses where to live; that is, a household maximizes its utility by simultaneously choosing a combination of commodities and a residential location.

An analysis of household location decisions depends on four key concepts:

Housing is measured in units of **housing services**, which provide a market-determined index of the size and quality of a housing unit.

The **price per unit of housing services** is the associated price concept per unit time, say per year.

The **rent** for a housing unit equals the price per unit of housing services multiplied by the number of units of housing services the unit contains. If the unit is an apartment, this rent is equivalent to the annual contract rent. If the unit is owner-occupied, this rent is not observed in the market place but is implicit. Note that this rent is not the same as land rent.

The **value** of a housing unit is the asset price associated with the net rental flow it generates. As in the case of land, this value is the present value of this rental flow.

In general, the more people are willing to pay for a house or apartment (holding its location constant), the more housing services that housing unit contains. In other words, housing services can be though of as an index that reflects the desirability to consumers of the characteristics of a particular housing unit. A formal way to express the concept is to say that the housing services provided by a particular housing unit are a function of the characteristics of that unit, or $H = f(X_1, X_2, X_3, \ldots , X_N)$, where f is a function that reflects consumer preferences and the X's are housing characteristics.

To simplify the discussion, one could think of size as the only housing characteristic people care about, so that housing services can be measured in square feet. This simplification is not necessary, however; for both conceptual and empirical purposes, any number of housing characteristics, including number of rooms, quality of construction, and style, can be incorporated into the housing services concept.

The price per unit of housing services is assumed to be constant at a given location; that is, each unit of housing services at a given location costs the same amount. As we will see, however, this price (and hence both rent and value) varies across locations in a systematic way. Indeed, the relationship between price per unit of housing services and location is the key to understanding the spatial dimension of housing markets. To bring location into the problem in a simple way, we will, as in the simple land model, express location as a function of distance from a central business district (CBD).

In symbols, let H stand for housing services, $P(u)$ for the price per unit of H at a location u miles from the CBD, $R(u)$ for housing rent at location u, and $V(u)$ for house value at location u. (Remember that land rent and land value always are written with an L subscript to distinguish them from housing rent and housing value, which for ease of presentation have no subscript.) Now the above discussion implies that

$$R(u) = P(u)\,H \tag{4}$$

and

$$V(u) = \sum_{t=1}^{T} \frac{R_t(u)}{(1+i)^t}\ , \tag{5}$$

where T is the expected lifetime of the building (in years).[5] If $R(u)$ is constant over time and T is greater than about 40, as it typically is for housing, then we can use the same simplification that we used for land; that is, we can write:

$$V(u) = \frac{P(u)\,H}{i} = \frac{R(u)}{i}\ . \tag{6}$$

For future reference, it is useful to note that equations (5) and (6) contain future values and therefore inevitably reflect expectations. One cannot write down the price of any asset, including housing, without considering what people expect rents and interest rates to be **in the future**. Strictly speaking, past and current rents and interest rates are not in the formula. In practice, we often implement these equations using current values for these variables, but this practice implicitly assumes that people have **static expectations**, that is, that they expect current values to continue indefinitely. This assumption need not be correct. In fact, we will discover many cases in which expected changes in rents or interest rates have a major impact on house values.

Before turning to housing bid functions, we must first make some assumptions about the nature of an urban area and about household location decisions. We will start with some very simple (and unrealistic) assumptions. Once the bid-function tool has been mastered, we can move on to more complex (and realistic) assumptions.

First, let us assume that all employment in an urban area is in the CBD, so that all workers commute to the CBD from their homes. Moreover, ignoring for now the complexities of an urban transportation system, let us assume that commuting distance is the straight-line distance between home and work and that commuting costs per mile are a constant, t. Finally, let us start by assuming that the price of housing services declines as one moves farther from the CBD. As we will see, this shape for $P(u)$ actually can be derived, but it is convenient to begin by assuming that it exists.

Now in deciding where to live, a household must compare the marginal benefits and marginal costs of moving farther from the CBD. The principal advantage of moving farther from the CBD is that the household encounters a lower price of housing. Let $\Delta P(u)$ be the change in the price of housing (per unit of H!) that occurs when a household moves from a location u to a location $u + \Delta u$. (The symbol stands for a change.) By assumption, $\Delta P(u)$ is always negative. In addition, remember that H is the amount of housing the household consumes. The amount of H varies from one location to another; for the moment, H should be interpreted as the consumption of housing at the starting location, u.) Thus the saving to the household from moving Δu miles farther from the CBD, which is the household's marginal benefit, is the drop in price per unit of housing multiplied by the quantity of housing the household consumes, or:

$$\text{savings} = -\Delta \, \mathrm{P}(u)\mathrm{H} \tag{7}$$

The disadvantage of moving farther from the CBD is that the household must spend more time and money to commute to work. Since *t* is the per mile cost of commuting, the increase in commuting costs from moving Δu miles farther from the CBD, which is the household's marginal cost, can be written as follows:

$$\text{increased cost} = t\Delta u \tag{8}$$

Now it should be clear that a household keeps moving farther from the CBD until the savings in the form of lower housing expenses are just offset by increased commuting costs; that is, until the marginal benefit from moving another mile just equals the marginal cost. In symbols, the household keeps moving until it finds a location, u^*, at which:

$$-\Delta\, P\left(u^*\right) H^* = t\Delta u \tag{9}$$

An example of this decision is given in Figure 2. Note that the marginal cost curve is horizontal; it always costs exactly $*t* to move one mile farther from the CBD.

Housing Bid Functions

The preceding pages describe how a single household selects a location when the function relating housing price to location, sometimes called the price-distance function, is given. In fact, however, the price-distance function is endogenous--that is, it is determined by the housing market. We now find the equilibrium price-distance function in an urban area in which all households are identical, which means that they have the same income and the same preferences. The assumption of identical households will be relaxed in later sections.

According to the logic just presented, each household keeps moving farther from the CBD until the net benefit from moving farther equals zero, that is, until the savings in the form of lower housing expenses are just offset by the increased commuting cost. With the assumption that all households are identical, however, all households will want to move to exactly the same location, labeled u^* in Figure 2. Of course this is impossible. All households cannot live the same distance from the CBD. Thus, the situation described in Figure 2 cannot be an equilibrium.

In general, an equilibrium is a situation in which no economic actor has an incentive to change his or her behavior. In this discussion, we are interested in **locational equilibrium**, which is defined as a situation in which no household has an incentive to move, that is, to change its residential location. How is locational equilibrium achieved? Because all households want to live at u^*, they compete with each other for the opportunity to live there, and the price of housing at u^*, namely $P(u^*)$, is bid up. Similarly, because nobody wants to live at other locations, the price of housing in other locations falls. Locational equilibrium is achieved when the price of housing has increased enough at u^* and decreased enough at other locations so that households do not care where they live.

In formal terms, locational equilibrium requires that all identical households achieve the same level of utility no matter where they live. If they could obtain more utility in another location, after all, they would have an incentive to move. Another way to put it is that locational equilibrium requires housing prices to adjust so that people who commute a long distance are compensated in the form of lower housing prices. This leads us to the notion of a housing bid function; this function indicates the maximum amount households are willing to pay for housing (that is, per unit H) at every location. The equilibrium $P(u)$ function often is called the **bid-price function**, to distinguish it from the bid function land, called the **bid-rent function**.(6)

In symbols, locational equilibrium requires that equation (9) be true at all inhabited locations. Written without the asterisk and rearranged to focus on the price of housing, this condition is

$$\frac{\Delta P(u)}{\Delta u} = -\frac{t}{H} \quad . \qquad \textbf{(10)}$$

In short, the price of housing must adjust so that this equation is satisfied everywhere; no matter where a household lives, moving a little farther from the CBD results in a decrease in housing costs that is exactly offset by an increase in commuting costs.

Thus equation (10) has two interpretations. First, if $P(u)$ is known, it can be interpreted as a single household's decision rule for selecting a location. Second, it can be interpreted as the market equilibrium condition that determines what $P(u)$ must look like. These two interpretations are perfectly consistent. If locational equilibrium has been achieved (the second interpretation), then prices are set such that a household's decision rule (the first interpretation) is satisfied at every single location.

This discussion implies that Figure 2 does not represent locational equilibrium for identical households. To achieve locational equilibrium, $P(u)$ must adjust so that the marginal benefit curve coincides with the marginal cost curve at all inhabited locations.

Note also that equation (10) reveals what the equilibrium $P(u)$ function, or bid-price function, looks like. The left side of this equation is the slope of this function, that is the change in $P(u)$--the rise--relative to any given change in *u*--the run. The price of housing declines with distance because locational equilibrium requires that households who live far from the CBD be compensated for their higher commuting costs in the form of lower housing prices.

As illustrated in Figure 3, the housing price drop required to compensate households for higher commuting costs is greater near the CBD than far from the CBD. To see why this is true, note that as the price of housing falls, households substitute housing for the composite consumption good. Thus, people who live far from the CBD consume more housing that people who live close to the CBD. According to equation (10), therefore, the value of t/H is smaller, and hence the slope of the bid function is flatter, the farther one moves from the CBD.

Another way to express this result is to say that, as shown by the condition for household equilibrium, equation (9), the compensation a household receives from moving farther from the CBD equals the drop in $P(u)$ multiplied by their housing consumption. Because people far from the CBD consume more housing, they require a smaller drop in housing prices to obtain a given level of compensation. In short, households' ability to substitute housing for other goods implies that the equilibrium bid-price function must have the shape illustrated in Figure 3. As in the case of land, therefore, substitution gives curvature to the bid function (but now it is substitution between housing and other goods in consumption, not between land and capital in production).

Finally, note that equation (10) specifies the slope of the bid-price function, but it does not specify its **intercept**, or, loosely speaking, its height. The intercept corresponds to the level of utility households achieve. The higher the intercept, the lower the utility level. In other words, holding income constant, a higher price for housing pivots inward a household's budget constraint and lowers the level of utility it can achieve. Because we do not know the intercept, we do not yet know what level of utility the households in an urban area will achieve. In fact, therefore, equation (10) actually specifies a family of bid functions, each with a different intercept, but does not indicate which member of the family represents equilibrium in the urban area.

The intercept of the bid function, and hence the utility level in the urban area, can be found in one of two ways. The first approach is called an **open model**. This approach assumes that each urban area is part of a system of urban areas and that people are free to move from one area to another. Locational equilibrium requires that households achieve the same utility level in all areas--otherwise they would move from one area to another. In this case, the appropriate utility level is the one that a household could obtain if it moved elsewhere. We will call this the **target utility level**. The equilibrium bid function is the one associated with this target utility level. (This approach is used in the computer program that forms the basis of your first assignment.)

The second approach is called a **closed model**. According to this approach, an urban area contains a fixed number of people and locational equilibrium requires that there be enough room for everyone. Remember that the lower the price of housing, the more housing people will buy. More housing requires more land. Thus, if the price of housing is too low, each household will consume "too much" land, and there will not be enough land in the urban area to satisfy all households' demand for housing. In this case, the intense household competition for space will boost the price of housing--that is, it will shift up the equilibrium bid-price function. The higher the bid-price function, the lower the level of utility households will achieve.

Of course, the size of the urban area is not fixed. If people want more land, they can expand into the agricultural land around an urban area. However, the farther a location is from the CBD, the lower the price of housing must be in order to compensate people for their high commuting costs. No matter how intense the competition for space, there exists some point, far from the CBD, at which the price of housing (and hence the underlying price of land) will drop so low that households cannot outbid farmers. We will return to this issue in Part III of these notes.

Before leaving basic housing analysis, let us briefly explore the link between the bid-price function and the residential bid-rent function. Because the demand for land is derived from the demand for housing, the residential bid-rent function can be derived from the residential bid-price function (and information about housing technology). People with no understanding of economics often get this line of causation backwards; they say that the price of housing is high in certain locations because land is expensive there. In fact, however, the price of land is high in certain locations because people are willing to pay a lot for housing there. Thus the shape of the residential bid-rent function is similar to the shape of the bid-price function. The only important difference in shape is that the residential bid-rent function is more curved than is the bid-price function because it reflects substitution in both production and consumption.

In short, bid functions provide a straightforward way to understand how housing and land prices vary within an urban area. As we will see in the next part, they also are crucial for understanding the allocation of different types of households to different locations. The conceptual foundations of bid functions have been strongly supported by the evidence; dozens of studies find a significant, negative relationship between housing prices and access to employment concentrations.[(7)]

Part II: Household Sorting and Neighborhood Amenities

A bid-price function describes the pattern of housing prices that would keep a **given type** of household in locational equilibrium. Every type of household has its own bid function. As shown by equation (10), the slope of a bid function for a particular type of household depends on their transportation costs per mile and on the amount of housing that each household wants to consume. Household types compete with each other for housing (and hence for land); the household type with the highest bid at a particular location wins the competition and therefore lives there. This allocation of household types to locations is called **sorting**.

Household Sorting

Sorting is a bit tricky because it depends on the slope, or steepness, of bid functions. As shown by Figure 4, households with **steeper** bid functions live closer to the CBD, and households with **flatter** bid functions live far from the CBD. In this figure, group 1, which has a steeper bid function, wins the competition for housing and land at all locations inside u^*, and group 2 wins the competition outside u^*. Hence u^* is the boundary between the residential zones of the two groups.

A steeper function corresponds to a function with a larger slope in absolute value. Hence the larger the ratio t/H, the steeper the function. In other words, households with the highest values of t/H live closest to the CBD. Another way to put this is that if you are trying to determine which of two household types lives closer to the CBD, the only thing you have to know is which type has the steepest bid function, that is, the highest value of t/H.

Knowing the slopes of the bid functions is not sufficient, however, for determining the boundary of the residential areas, u^* in Figure 4. To determine this boundary you also must know the heights of all the relevant bid functions. As mentioned in Part I of these notes (and as pursued in Part III), these heights are related to utility levels, with higher heights corresponding to lower utility levels.

One key question to ask is whether high- or low-income households have steeper bid functions. The answer to this question determines whether high- or low-income households live closer to the CBD. Remember that the slope of a bid function equals $-t/H$. This question is equivalent to asking whether this ratio increases or decreases with income. If it decreases with income, that is, if high-income people have lower values of t/H than do low-income people, then high-income people will live farther from the CBD.

The impact of income on the slope of a bid function is complicated because income affects both t and H. Higher-income commuters may have access to relatively low-cost modes of transportation, such as fuel-efficient cars or faster highways, but they also have a higher opportunity cost for their time. Most studies indicate that the latter effect is larger, so that t increases with income. In addition, housing is a normal good, so that high-income people consume more H than do low-income people; that is, H increases with income. Because both t and H increase with income, the impact of income on the ratio of t to H is ambiguous; one cannot determine on conceptual grounds alone whether high- or low-income people live farther from the CBD.

To be more precise, the impact of income on the slope of a bid function depends on whether income has a larger proportional impact on t or on H. Suppose income goes up by 1 percent. Then the income elasticity of demand for housing, e_H, determines how much H will go up. The income elasticity, you will recall, is the percentage change in consumption for any percentage change in income. Similarly, the income elasticity of per-mile transportation costs, e_t, indicates how much t goes up with income. The bid function flattens with income, and hence higher-income people live farther from the CBD, if the income elasticity of demand for housing is greater than the income elasticity of per mile transportation costs. In formal terms, the condition for "normal" sorting, with higher-income people farther from the CBD is:

$$e_H = \frac{\Delta H/H}{\Delta Y/Y} > e_t = \frac{\Delta t/t}{\Delta Y/Y} \quad (11)$$

Most scholars agree that the income elasticity of demand for housing is greater than the income elasticity of per-mile commuting costs, so that the condition for normal sorting is met.[8]

Bid functions therefore provide a simple explanation for a key stylized fact about urban areas in the United States: high-income people are concentrated in the suburbs and low-income people are concentrated in the city. Moreover, and this is a crucial point for this course, the simple analytics of bid functions helps to explain why poverty is concentrated in certain neighborhoods.

Some people are puzzled by this logic. How, they ask, can low-income people ever outbid high-income people for housing. The key to understanding this issue is to remember that bids are expressed per unit of housing services. Hence low-income people win the competition because they bid a high amount **per unit of housing services** but consume a relatively low quantity of housing services. Take the simple example in which housing services are measured in square feet. Then low-income people select small apartments or double up so that they consume very few square feet per capita. Inside u^* in Figure 4 (and with group 1 defined as low-income), low-income people bid more than high-income people per square foot, so a landlord can make more money by renting to low-income people who double up than to high-income people who do not.

Sorting results also can be derived when the analysis is extended to employment outside the CBD. Figure 5 considers the case of one suburban employment location. The bid functions of suburban workers are highest at their point of employment and the bid functions of city workers are highest at the CBD. City workers bid the most for housing inside u^* and therefore win the competition for housing at the more central locations. If central city workers have much flatter bid functions than suburban workers, due, say, to relatively high incomes, then it is possible that their bid functions also are higher than those of suburban workers at locations far from the CBD. In Figure 5, city workers win the competition for housing outside u^+.

Finally, sorting concepts can be applied to the competition between residential and non-residential land uses. Remember that there is a bid-rent function associated with any bid-price function. Land is allocated according to bid-rent functions; the highest bid-rent at a given location wins the competition for land there. Firms often have steep bid-rent functions near employment concentrations, either because they value access to a transportation node or because they value access to other firms. In this case, firms often have the steepest bid-rent functions close to employment concentrations and therefore win the competition for land there. Indeed, if this were not the case, employment concentrations would not exist!

Neighborhood Amenities

One important extension of these bid-function ideas is to neighborhood amenities, such as the quality of surrounding houses, air pollution, access to parks, the crime rate, and so on. This extension has implications for many different types of urban policy.

Within a household type, households must be compensated for living in a neighborhood with poor amenities. Hence housing bids are a function of neighborhood amenities, as well as of access to employment. If A measures an amenity, then we can write a bid function as $P(A, u)$. If we are looking at a desirable amenity, then P increases with A. This case is drawn in Figure 6. This figure holds u constant (as indicated by the asterisk on the u argument); that is, it considers neighborhoods with different levels of A but the same access to employment.

This application of bid functions to neighborhood amenities is strongly supported by extensive empirical work in urban economics. The price of housing, controlling for structural housing characteristics, is higher in neighborhoods with more desirable amenities.[9]

Neighborhood amenities also influence household sorting. The key issue here is which type of household has a greater willingness to pay for an increment in the amenity. A stronger willingness to pay for this **increment** translates into a steeper bid function in Figure 6. As shown in Figure 7, the household type with the steeper bid function is sorted into the locations with the higher levels of the neighborhood amenity. For most amenities, high-income households are willing to pay more for an increment in the amenity than are low-income households. In general, therefore, high-income households sort into locations with better amenities.

This result reinforces the sorting result in the previous section. Because suburban areas tend to have better amenities, the sorting based on access to employment alone is reinforced and magnified by the sorting based on amenities. Competition in the housing market therefore sorts higher-income households into nicer neighborhoods--farther from employment concentrations.

It is important to note that competition in the housing market, not zoning, is the primary source of sorting in American urban areas. Large-lot and other types of residential zoning sometimes have an impact on household locations when they prevent a change in sorting that might occur over time (about which more later). Zoning is not needed, however, to produce sorting in the first place. Moreover, most large-lot zoning simply formalizes a result that occurs naturally in the market place.

One analytical complication in all this is that amenities include the characteristics of the people in a neighborhood. We will encounter some examples of this later in the class. In this case, bid function analysis is much more complicated because neighborhood amenities depend on sorting and sorting depends on neighborhood amenities. This two-way causation is difficult to analyze. This complexity does not, however, alter the statement that high-income people tend to sort into the most desirable neighborhoods.

Part III: Neighborhood Change and Urban Equilibrium

Bid functions describe locational equilibrium in an urban area; that is, they describe a situation in which no one has an incentive to move. In many cases, however, we will be concerned with change, particularly neighborhood change, instead of with equilibrium. As it turns out, bid functions also shed light on neighborhood change, which can be seen as a movement from one equilibrium to another.

Basic Analysis of Neighborhood Change

To understand neighborhood change, we must return to the issue of bid function **height**. Remember that the equations for locational equilibrium define a family of bid functions, each one of which corresponds to a different utility level for the associated household type. The higher the bid function, the lower the utility level. Another way to put it is that the height of a bid function reflects the intensity of the competition for space facing the members of a household type. If space is scarce relative to demand for it, so that the competition for space is intense, the bid function gets pushed up (and the utility level gets pushed down). If space is relatively plentiful, on the other hand, the bid function need not be very high. These results can be seen as simple applications of basic supply and demand: the higher the demand curve relative to the supply curve, the higher the equilibrium price.

This brief analysis of height gives us enough intuition to start analyzing neighborhood change. Suppose something happens to alter the housing demand for some type of household. One important example is immigration into a city, which boosts housing demand. As people move into a city, they compete with households already there for the existing housing. This competition drives up bid functions. Suppose the immigrants have low incomes. Then as shown in Figure 8, the upward shift in the low-income bid function shifts the boundary between low- and high-income neighborhoods from u_1 to u_2. This is an example of neighborhood change.

The change in Figure 8, which concerns the price per unit of housing services, is accompanied by changes in the housing units themselves, that is, in the housing services they contain. Low-income people cannot spend as much on housing as high-income people. The only way they win the competition for space in this changing neighborhood is by bidding more per unit of housing services and accepting much smaller (and otherwise less desirable) units. Thus landlords in this changing neighborhood **convert** their apartments from large units to small units or else they allow low-income families to double up in units previously reserved for a single family.

Out migration also shifts bid functions--but downward. This case is illustrated by reversing the arrow in Figure 8. If low-income people leave an urban area for better opportunities elsewhere, the competition for space in low-income neighborhoods drops, the low-income bid function falls, and some neighborhoods change from low to high income. This change requires upward conversion; walls are knocked down to turn small apartments into large ones. This type of change might be limited by neighborhood amenities, which are not considered in Figure 8. As discussed earlier, high-income people sort into neighborhoods with desirable amenities. If the low-income neighborhoods have very poor amenities (as is likely with many low-income people moving out, rents dropping, and perhaps, buildings being abandoned), the change in household allocation in Figure 9 may not take place; instead, the price of housing and the quality of the neighborhood simply drop for low-income people who remain there.

Low-income neighborhoods also may disappear if the number of middle- or high-income households increased. One important example of this phenomenon is the baby boom, which boosted the number of young adults, many with well-paying jobs, in cities. This case, which is shown in Figure 9, involves an increase in the high-income bid function and, as in a reversed Figure 8, an expansion of the high-income area. This case is an example of **gentrification**.

Precise Analysis of Neighborhood Change: Open and Closed Models

These examples give the correct intuition for neighborhood change, but they are not the whole story. The problem is that determining the exact height of a bid function after something has changed is a complex analytical problem. Let us now cover this problem with a bit more precision.

As explained earlier, there are two broad approaches to finding bid-function heights. One approach is called an **open model**. According to this approach, the urban area under investigation is part of a system of urban areas and people can migrate from one urban area to another. In this case, a household's utility must be set at the level it can attain in other urban areas. The height of the bid functions is the height associated with this system-wide or target utility level. This approach is appropriate when one is analyzing changes in a single urban area that is part of a system of urban areas. It has the advantage that it is easier to solve (and hence is built into the Gridcity program) but the disadvantage that it relies on the abstract concept of a utility level.

The alternative approach is called a **closed model**, which assumes that the population of an urban area is fixed. This approach is appropriate for a single urban area when migration across areas is not possible. It also is appropriate for analyzing changes that affect all urban areas at the same time. After all, when an event affect all urban areas, nobody has an incentive to migrate from one urban area to avoid it (or to take advantage of it). This approach does not require any abstract concepts, but it is quite complex algebraically. (These notes do not attempt to demonstrate this complexity!)

Now suppose we want to determine the height of a particular household type's bid function in an closed model. This height must be set so that there is enough room for this household type. As the height of the bid function increases, the room available for the household type increases in two ways. First, it increases because the household type outbids other household types with which it has boundaries; that is, the number of neighborhoods in which the household lives expands. This effect is illustrated in Figures 8 and 9.

Second, an increase in the height of a household type's bid function leads to greater population density for that household type. This link to density is a bit tricky to understand but crucial for full understanding of bid functions. When the price of housing goes up at a given location (that is, when the bid function shifts upward), people substitute away from housing toward other goods. In other words, an increase in housing price leads people to buy smaller houses or apartments. This behavior is, of course, an example of the most basic principle of micro-economics, namely that people substitute away from any commodity whose price goes up. As a result of this substitution, more people can fit onto the same land area.

Furthermore, the increase in the price of housing results in an increase in the derived demand for land and hence in the equilibrium price of land. As the price of land increases, housing suppliers substitute capital for land, at least in the long run. This substitution in inputs magnifies the effect of substitution in consumption; not only do people buy smaller houses, but these smaller houses are build with a smaller "footprint." Builders might supply 1000 square-foot town houses on two floors instead of on one, for example.

The first application of these ideas is to the overall size of an urban area. An urban area expands until the land rent associated with housing equals the agricultural rental rate, R_A. Beyond this point, housing cannot compete land away from agriculture. So what is the right height for the bid-rent function in a closed model? It is the height that provides enough room for the population of the city. As illustrated in panel A of Figure 10, an increase in this height results in an expansion in the size of the city from u_1 to u_2 and thereby makes room for more people. Although not shown in this figure, this change also increases population density at every location in the city. To find the equilibrium height, therefore, one must keep increasing the bid-rent function until the physical size and density of the urban area are high enough to fit in the required number of people. Panel B shows that this same analysis can be conducted with a bid-price function if we use the concept of the opportunity cost of pulling resources out of agriculture into housing, labeled P_A. (Please note that this discussion is not intended to suggest that bid functions shift around in this manner over time in a search for equilibrium. Instead, this discussion simply describes how an analyst can figure out where the equilibrium is.)

Now we can return to the examples presented earlier. Consider the case of Figure 8. Suppose the upward shift in the low-income bid function is sufficient to make room for all the immigrants. In other words, the expansion in the area inhabited by low-income people and the increase in low-income population densities provides the necessary room. The problem is that this change knocks high-income households out of equilibrium--there is no longer enough room for them. Hence, as shown in Figure 11, the high-income bid function shifts up, knocking low-income households out of equilibrium. Hence the low-income bid function must go up a bit more, driving up low-income densities. In the new equilibrium, the low-income bid function (and corresponding density function) is higher than shown in Figure 8, the low-income area has expanded less than in Figure 8, and the high-income bid function (and density function) has shifted up. Now you can see why Figure 8 (or 9) does not provide the whole story!

Long-Term Changes in Urban Residential Structure

These tools also can be used to help understand how urban areas change in response to systematic changes in the factors that influence household behavior, such as transportation costs and income. Remember from equation (11) that the slope of a bid-price function equals $-t/H$. When t drops, due to the building of a new highway system or the introduction of fuel-efficient cars, for example, then the slope of the bid function also drops. If this change happens in all urban areas, then this change must be analyzed with a closed model. In Figure 12, simply flattening the bid function without changing its intercept both increases population density everywhere and increases the size of the urban area. Unless the intercept is dropped, therefore, the urban area has far too much room for its population. As the bid function drops, the physical size drops and so does population density until there is just the right amount of room. The drop in t therefore results in lower population densities at central locations and higher population densities far from the CBD.

An increase in income works the same way. As income increase, people consume more housing services. Moreover, an increase in H results in a drop in the slope of the bid function. Figure 12 applies directly to this case, as well. Because bid functions are directly linked to population densities, it follows that declines in transportation costs and increases in income are major sources of suburbanization. Either of these changes shifts urban population away from the city center toward the suburbs. Note also that increases in transportation costs or decreases in income work in exactly the opposite direction.

Endnotes

[1] This discussion assumes that you are familiar with present value and discounting. You may want to review your notes on these topics.

[2] This simplification can be derived with a bit of algebra. Just subtract $V_L(1+i)$ from V_L.

[3] If the "something" produced on land is hiking or land preservation, the distinction between land and the thing being produced is abstract and perhaps not helpful. In the urban context, however, the "something" we are talking about is housing or a manufacturing plant or a retail establishment, where the distinction between land and output is clear.

[4] This assumption defines what is called a fixed-coefficients or Leontief production function.

[5] By writing H as a function of housing characteristics, equation (4) can be estimated for the rental market and equation (5) below can be estimated for single-family houses. Indeed, dozens of studies use this framework to examine housing markets. Equations of this type are called hedonic equations.

[6] Yes, this terminology is a bit confusing because rents appear in both the land and housing market. But urban economic models do not focus on housing rents so this terminology is widely accepted among urban economists.

[7] For a recent survey of this evidence, see N.E. Coulson, "Tests of the Monocentric Model," *Land Economics*, August 1991, pp. 299-307.

[8] Some studies find that the two elasticities are fairly close in magnitude, however, so that higher-income people might not have flatter bid functions in all circumstances.

[9] For a recent review of this evidence, see T.J. Bartik and V. K. Smith, "Urban Amenities and Public Policy," In the *Handbook of Urban and Regional Economics*, Vol. 2, edited by E.S. Mills (North-Holland, 1987), pp. 1207-1254.